9th Class 2016

Math (Science)	Group-I	Paper-I
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any Six (6) questions: 12

(i) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$, verify that $A + B = B + A$.

Ans L.H.S

$$A + B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

R.H.S

$$B + A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1+2 & 1+3 \\ 2+2 & -2+3 & 2+1 \\ 3+1 & 1-1 & 3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

L.H.S = R.H.S

(ii) If $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$, $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, find AB.

Ans

$$AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3(6) + 0(5) \\ -1(6) + 2(5) \end{bmatrix}$$

$$= \begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(iii) Simplify by using rule of exponent: $\left(\frac{8}{125}\right)^{-4/3}$.

Ans $\left(\frac{8}{125}\right)^{-4/3} = \frac{1}{\left(\frac{8}{25}\right)^{4/3}} = \left(\frac{125}{8}\right)^{4/3}$

$$= \frac{(125)^{4/3}}{(8)^{4/3}}$$

$$= \frac{(5^3)^{4/3}}{(2^3)^{4/3}}$$

$$= \frac{5^{3 \times 4/3}}{2^{3 \times 4/3}}$$

$$= \frac{5^4}{2^4}$$

$$= \frac{625}{16}$$

(iv) Simplify and write your answer in the form of a + bi : $\frac{-2}{1 + i}$.

Ans $\frac{-2}{1+i} = \frac{-2}{1+i} \times \frac{1-i}{1-i}$

$$= \frac{-2(1-i)}{1-i^2}$$

$$= \frac{-2+2i}{1-(-1)}$$

$$= \frac{-2+2i}{2}$$

$$= \frac{-2}{2} + \frac{2i}{2}$$

$= -1 + i$

(v) Find the value of x: $\log_{81} 9 = x$.

Ans

$$\log_{81} 9 = x$$
$$81^x = 9$$
$$(9^2)^x = 9$$
$$9^{2x} = 9^1$$
$$\Rightarrow \quad 2x = 1$$
$$x = \frac{1}{2}$$

(vi) Express as a single logarithm:

$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$.

Ans

$$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$$
$$= \log x - \log x^2 + \log(x+1)^3 - \log(x^2-1)$$
$$= \log\frac{x(x+1)^3}{x^2(x^2-1)} = \log\frac{(x+1)^3}{x(x+1)(x-1)}$$
$$= \frac{\log(x+1)^2}{x(x-1)}$$

(vii) Reduce the following rational expression to th lowest form: $\dfrac{x^2-4x+4}{2x^2-8}$.

Ans

$$\frac{x^2-4x+4}{2x^2-8} = \frac{x^2-2x-2x+4}{2(x^2-4)}$$
$$= \frac{x(x-2)-2(x-2)}{2(x^2-2^2)}$$
$$= \frac{(x-2)\cancel{(x-2)}}{2(x+2)\cancel{(x-2)}}$$
$$= \frac{x-2}{2(x+2)}$$

(viii) If $x = 4-\sqrt{17}$, then find $\dfrac{1}{x}$.

Ans

$$x = 4-\sqrt{17}$$
$$\frac{1}{x} = \frac{1}{4-\sqrt{17}}$$
$$= \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$

$$= \frac{4+\sqrt{17}}{(4)^2-(\sqrt{17})^2}$$

$$= \frac{4+\sqrt{17}}{16-17}$$

$$= \frac{4+\sqrt{17}}{-1}$$

$$\frac{1}{x} = -4-\sqrt{17}$$

(ix) **Factorize:** $x^2 - y^2 - 4x - 2y + 3.$

Ans By adding and subtracting 1.

$$x^2 - y^2 - 4x - 2y + 4 - 1$$
$$= x^2 - 4x + 4 - y^2 - 2y - 1$$
$$= (x-2)^2 - (y^2 + 2y + 1)$$
$$= (x-2)^2 - (y+1)^2$$
$$= (x - 2 + y + 1)(x - 2 - y - 1)$$
$$= (x + y - 1)(x - y - 3)$$

3. Write short answers to any Six (6) questions: 12

(i) **Find H.C.F. of the polynomials by factorization:**

$x^2 - 4,\ x^2 + 4x + 4,\ 2x^2 + x - 6.$

Ans

$$x^2 - 4 = (x)^2 - (2)^2$$
$$= (x+2)(x-2)$$
$$x^2 + 4x + 4 = x^2 + 2x + 2x + 4$$
$$= x(x+2) + 2(x+2)$$
$$= (x+2)(x+2)$$
$$2x^2 + x - 6 = 2x^2 + 4x - 3x - 6$$
$$= 2x(x+2) - 3(x+2)$$
$$= (x+2)(2x-3)$$

Hence H.C.F $= (x+2)$

(ii) **Solve:** $\frac{3}{y-1} - 2 = \frac{3y}{y-1},\quad y \neq 1.$

Ans

$$\frac{3}{y-1} - 2 = \frac{3y}{y-1}$$

$$(y-1) \times \frac{3}{y-1} - 2(y-1) = (y-1) \times \frac{3y}{y-1}$$

$$3 - 2y + 2 = 3y$$
$$-2y + 5 = 3y$$

$$3y + 2y = 5$$
$$5y = 5$$
$$y = 1$$

(iii) Find the solution set of: $|2x + 3| = 11$.

Ans $|2x + 3| = 11$

$2x + 3 = 11$; $2x + 3 = -11$

$2x = 11 - 3$; $2x = -11 - 3$

$2x = 8$; $2x = -14$

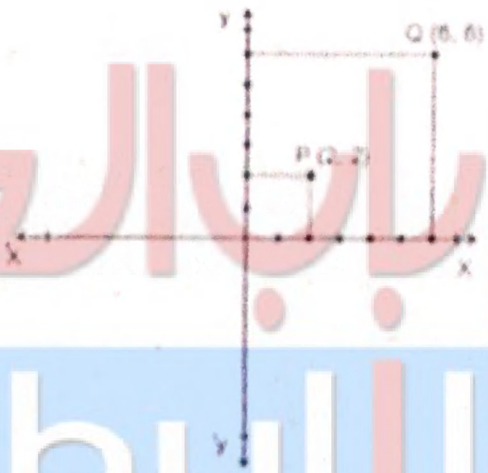
$x = 4$; $x = -7$

(iv) Define Cartesian Plane.

Ans The Cartesian plane establishes (one-to-one) correspondence between the set of ordered pairs $R \times R = \{(x, y) \mid x, y \in R\}$ and the points of the Cartesian plane.

(v) Plot the points on quadrant: P(2, 2), Q(6, 6)

Ans



(vi) Define scalene triangle.

Ans A triangle is called a scalene triangle if measures of all the three sides are different.

(vii) Find the distance between the points: A(9, 2), B(7, 2).

Ans
$$|AB| = \sqrt{(7-9)^2 + (2-2)^2}$$
$$= \sqrt{(-2)^2 + 0}$$
$$= \sqrt{4}$$
$$|AB| = 2$$

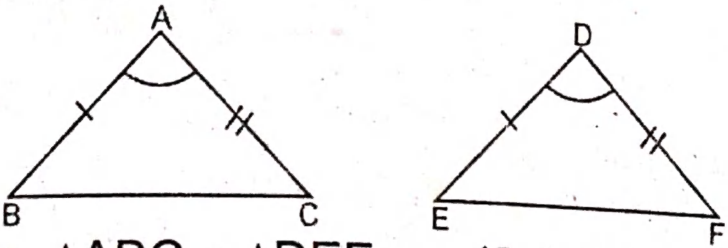
(viii) What do you mean by SAS ≅ SAS?

Ans In any correspondence of two triangles, if two sides and their included angle of one triangle are congruent to

the corresponding two sides and their included angle of the other triangle, then the triangles are congruent.

In $\Delta ABC \longleftrightarrow \Delta DEF$, shown in the following figures,

$$\text{if} \begin{cases} \overline{AB} \cong \overline{DE} \\ \angle A \cong \angle D \\ \overline{AC} \cong \overline{DF} \end{cases}$$



then $\Delta ABC \cong \Delta DEF$ (S.A.S Postulate)

(ix) Define parallelogram.

Ans A figure formed by four non-collinear points in the plane is called parallelogram. Its characteristics are as under:

1. Its equal opposite sides are of equal measure.
2. Its opposite sides are parallel.
3. Measure of none of the angle is 90^{o}.

4. Write short answers to any Six (6) questions: 12

(i) Where do the right bisectors of the sides of an acute triangle and right triangle intersect each other?

Ans The right bisectors of the sides of an acute triangle intersect each other inside the triangle. The right bisectors of the sides of a right triangle intersect each other on the hypotenuse.

(ii) If 3 cm and 4 cm are lengths of two sides of a right angled triangle, then what should be the third side (hypotenuse) length of the triangle?

Ans

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$
$$(\text{Hypotenuse})^2 = (3)^2 + (4)^2$$
$$(\text{Hypotenuse})^2 = 9 + 16$$
$$(\text{Hypotenuse})^2 = 25$$
$$\sqrt{(\text{Hypotenuse})^2} = \sqrt{25}$$
$$(\text{Hypotenuse}) = 5 \text{ cm}$$

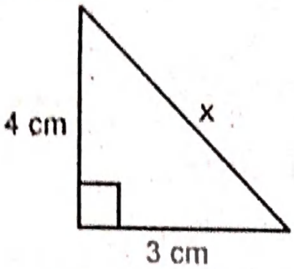
(iii) **What is difference between a line and plane?**

Ans For the sake of plane, two mutually perpendicul straight lines are drawn. But in that plane, we get a lin after joining two points.

(iv) **Define Pythagoras theorem.**

Ans In a right angled triangle, the square of the length hypotenuse is equal to the sum of the squares of th lengths of the other two sides.

(v) **Find the value of x:**



Ans $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$$x^2 = 4^2 + 3^2$$
$$x^2 = 16 + 9$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = 5 \text{ cm}$$

(vi) **Verify that following measures are the sides o right-angled triangle:**

$a = 5$ cm, $b = 12$ cm, $c = 13$ cm

Ans $(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$$(13)^2 = 5^2 + 12^2$$
$$169 = 25 + 144$$
$$169 = 169$$

(vii) **Define the rectangular region.**

Ans A rectangular region is the union of a rectangle and its interior.

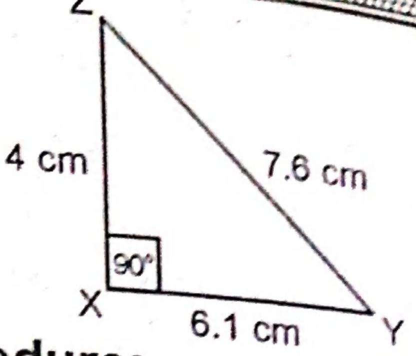
(viii) **Define the incentre of a triangle.**

Ans The internal bisectors of the angles of a triangle meet at a point called the incentre of the triangle.

(ix) **Construct a triangle XYZ, in which:**

$$m\angle X = 90^\circ, \; m\overline{XY} = 6.1 \text{ cm}, \; m\overline{YZ} = 7.6 \text{ cm}$$

Ans



Constructive Procedure:

1. Draw a line XY = 6.1 cm.
2. At point X, draw 90° angle.
3. At point Y, draw 7.6 cm long arc on Z.
4. Join Y and Z.
5. ΔXYZ is our required triangle.

(Part-II)

NOTE: Attempt Three (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the given system of linear equation by Cramer's rule: $3x - 2y = -6$ **(4)**

$5x - 2y = -10$

Ans

$$3x - 2y = -6$$
$$5x - 2y = -10$$

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$$

$$|A| = -6 + 10$$
$$= 4$$

$$A_x = \begin{bmatrix} -6 & -2 \\ -10 & -2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 3 & -6 \\ 5 & -10 \end{bmatrix}$$

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} -6 & -2 \\ -10 & -2 \end{vmatrix}}{4}$$

$$x = \frac{-6x(-2) - (-10 \times -2)}{4}$$

$$= \frac{12-20}{4} = -\frac{8}{4} = -2$$

$$y = \frac{|Ay|}{|A|} = \frac{\begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}}{4}$$

$$y = \frac{-30+30}{4} = \frac{0}{4} = 0$$

$$y = 0$$

(b) Use laws of exponents to simplify: (4)

$$\frac{(81)^n \times 3^5 - (3)^{4n-1}(243)}{(9^{2n})(3^3)}$$

Ans

$$\frac{(81)^n \times 3^5 - (3)^{4n-1}(243)}{(9^{2n})(3^3)}$$

$$= \frac{(3^4)^n \times 3^5 - 3^{4n-1} \times 3^5}{(3^{2n})^2 \times 3^3}$$

$$= \frac{3^{4n+5} - 3^{4n-1+5}}{3^{4n} \times 3^3}$$

$$= \frac{3^{4n+5} - 3^{4n+4}}{3^{4n+3}}$$

$$= \frac{3^{4n+4} \times 3 - 3^{4n+4}}{3^{4n+3}}$$

$$= \frac{3^{4n+4}(3-1)}{3^{4n+3}}$$

$$= 3^{4n+4-4n-3} \times 2$$

$$= 3^1 \times 2$$

$$= 6$$

Q.6.(a) Use log tables to find the value of: (4)

$$\frac{(1.23)(0.6975)}{(0.0075)(1278)}$$

Ans

$$x = \frac{(1.23)(0.6975)}{(0.0075)(1278)}$$

By taking log

$$\log x = \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$$

$$\log x = 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$$

$$\log x = 0.0899 - 1 + .8435 + 3 - .8751 - 3.1065$$

$= -1.0482$

$= -1.0482 - 2 + 2$

$\log x = \bar{2}.9518$

Taking antilog

$x = 0.0895$

(b) If $x^2 + y^2 + z^2 = 78$ and $xy + yz + zx = 59$, then find the value of $x + y + z$. (4)

Ans

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$
$$= 78 + 2(59)$$
$$= 78 + 118$$
$$= 196$$
$$(x + y + z)^2 = 14^2$$

Taking underroot both sides

$$\sqrt{(x + y + z)^2} = \pm\sqrt{14^2}$$
$$x + y + z = \pm 14$$

Q.7.(a) Factorize: $64x^3 + 27y^3$ (4)

Ans

$$64x^3 + 27y^2$$
$$(4x)^3 + (3y)^2$$
$$= (4x + 3y)((4x)^2 + (3y)^2 - 4x \times 3y)$$
$$= (4x + 3y)(16x^2 + 9y^2 - 12xy)$$

(b) For what value of k is $(x + 4)$ the H.C.F of $x^2 + x - (2k + 2)$ and $2x^2 + kx - 12$? (4)

Ans

$$2x^2 + kx - 12$$
$$P(x) = 2x^2 + kx - 12$$

As $x + 4 = 0 \Rightarrow x = -4$

$$P(-4) = 2(-4)^2 + k \times (-4) - 12$$
$$= 32 - 4k - 12$$

P must be zero

$$0 = 20 - 4k$$
$$20 = 4k$$
$$5 = \frac{20}{4} = k$$
$$q(x) = x^2 + x - 2k - 2$$
$$q(-4) = (-4)^2 + (-4) - 2k - 2$$
$$= 16 - 4 - 2k - 2$$
$$= 10 - 2k$$

R must be zero

$$10 = 2k$$

$$\frac{\cancel{10}}{\cancel{2}} = k$$

$$5 = k$$

Q.8.(a) Find the solution set of the equation:

$$\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}, \; x \neq -2$$

Ans

$$\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}$$

$$\frac{2}{3x+6} + \frac{1}{2x+4} = \frac{1}{6}$$

$$\frac{2}{3(x+2)} + \frac{1}{2(x+2)} = \frac{1}{6}$$

$$\frac{4+3}{6(x+2)} = \frac{1}{6}$$

$$\frac{7}{6(x+2)} = \frac{1}{6}$$

$$6x + 12 = 42$$

$$6x = 42 - 12$$

$$6x = 30$$

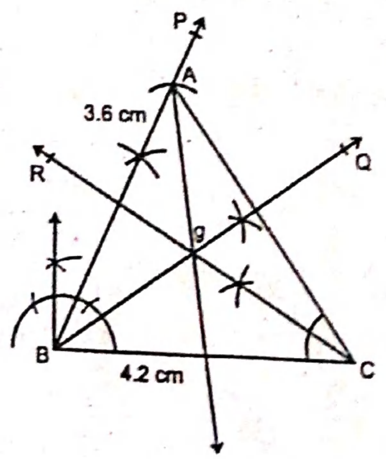
$$x = \frac{30}{6}$$

$$x = 5$$

$$\{5\}$$

(b) Construct Δ ABC, draw the bisectors of its angle and verify their concurrency:

Ans $m\overline{AB} = 3.6$ cm, $m\overline{BC} = 4.2$ cm and $m\angle B = 75^\circ$

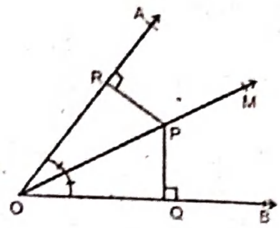


Constructive Procedure:

1. Take a line $\overline{BC} = 4.2$ cm.
2. Make an angle $\angle CBP = 75^o$.
3. Draw an arc of 3.6 cm at B which cuts $\overrightarrow{BP}$ at A.
4. Join A and C.
5. ABC is the required triangle.
6. Take bisectors of angles as $\overrightarrow{AP}$, $\overrightarrow{BQ}$ and $\overrightarrow{CR}$.

Q.9. Prove that any point on the bisector of an angle is equidistant from its arms. (4)

Ans



Given: A point P is on $\overrightarrow{OM}$, the bisector of $\angle AOB$.

To prove: $\overline{PQ} \cong \overline{PR}$ *i.e.*, P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Construction: Draw $\overrightarrow{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$.

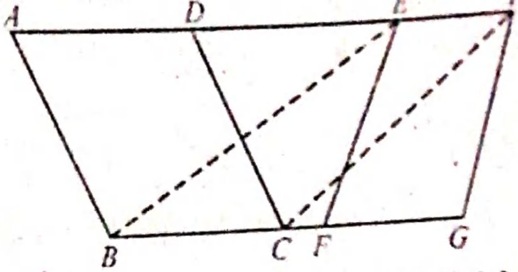
Proof:

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PQO \cong \angle PRO$	Construction
$\angle POQ \cong \angle POR$	Given
$\Delta POQ \cong \Delta POR$	S.A.A ≅ S.A.A
Hence $\overline{PQ} \cong \overline{PR}$	(corresponding sides of congruent triangles)

OR

Prove that parallelogram on equal bases and having the same (or equal) altitudes are equal in area.

Ans



Given: Parallelogram ABCD, EFGH are on equal base $\overline{BC}$ and $\overline{FG}$, having equal altitudes.

To prove: Area of (parallelogram ABCD) = Area of ($||^{gm}$ EFGH)

Construction: Place the parallelograms ABCD and EFGH so that their equal bases $\overline{BC}$, $\overline{FG}$ are in the straight line BCFG. Join $\overline{BE}$ and $\overline{CH}$.

Proof:

Statements	Reasons
The given $||^{gm}$ ABCD and EFGH are between the same parallels. Hence ADEH is straight line $||$ to $\overline{BC}$	Their altitudes are equal. (Given)
$m\overline{BC} = m\overline{FG}$	Given
$= m\overline{EH}$	EFGH is a parallelogram.
Now $m\overline{BC} = m\overline{EH}$ and they are parallel.	
$\overline{BE}$ and $\overline{CH}$ are both equal and parallel. Hence, EBCH is a parallelogram.	A quadrilateral with two opposite sides congruent and parallel is a parallelogram.
Now $||^{gm}$ ABCD = $||^{gm}$ EBCH (i)	Being on the same base $\overline{BC}$ and between the same parallels.
But $||^{gm}$ EBCH = $||^{gm}$ EFGH (ii)	Being on the same base $\overline{EH}$ and between the same parallels.
Hence, area ($||^{gm}$ ABCD) = Area (($||^{gm}$ EFGH)	From (i) and (ii)